Rational Root Thm

Rational ↑ can be written as a fraction

Root ↑ x-int, soln' zeros

- If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_0$

  a_n ↑ leading coeff. a_0 ↑ constant

  has rational roots, they are of the form $\frac{p}{q}$

  p's : factors of constant

  q's : factors of leading coeff.

- A polynomial of degree n has <u>at most</u> n real roots.

- Imaginary roots + irrational roots come in pairs. $a+bi \rightarrow a-bi$

  $-b \pm \sqrt{}$

Ex: List the possible rational roots

$$f(x) = 2x^5 + 3x^3 + 4x^2 - 6$$

6's (p) ← p's

(q) ~~25~~

	±1	±2	±3	±6
±1	±1/1 = ±1	±2	±3	±6
±2	±1/2	±2/2 = ±1	±3/2	±6/2 = ±3

±1, ±2, ±3, ±6, ±1/2, ±3/2

Ex: Factor completely. List all x-int, graph.

$$p(x) = x^3 - 7x^2 + 14x - 8$$

$\uparrow$ Q's $\qquad$ $\uparrow$ P's

Optional Step 1: make your P/Q's list

	P's			
	±1	±2	±4	±8
Q's ±1	±1	±2	±4	±8

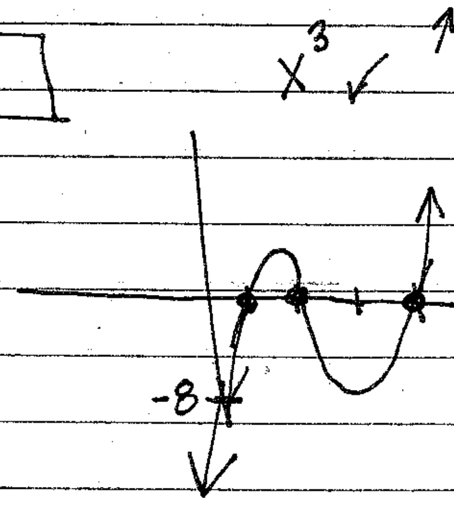
Step 2: Look for a factor:
↳ Synthetic ÷ + look for remainders = 0.

$$\begin{array}{c|cccc} 1 & 1 & -7 & 14 & -8 \\ & & 1 & -6 & 8 \\ \hline & 1 & -6 & 8 & \cancel{0} \checkmark \end{array}$$

$(x-1)(x^2-6x+8)$

$$\boxed{(x-1)(x-2)(x-4)}$$

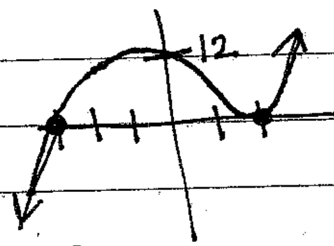
x-int: 1, 2, 4



Ex: $p(x) = x^3 - x^2 - 8x + 12$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -8 & 12 \\ & & 1 & 0 & -8 \\ \hline & 1 & 0 & -8 & \end{array} \qquad \begin{array}{r|rrrr} -1 & 1 & -1 & -8 & 12 \\ & & -1 & 2 & 6 \\ \hline & 1 & -2 & -6 & \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -8 & 12 \\ & & 2 & 2 & -12 \\ \hline & 1 & 1 & -6 & 0 \end{array} \quad \left\{ \begin{array}{l} (x-2)(x^2+x-6) \\ (x-2)(x+3)(x-2) \\ (x+3)(x-2)^2 \end{array} \right.$$



degree $= 3$

$\swarrow \quad \nearrow$

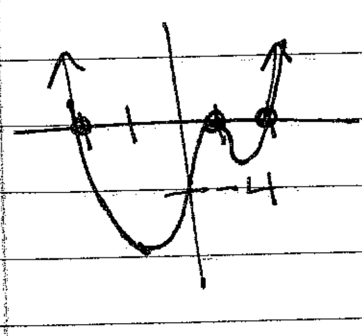
Ex: $p(x) = x^4 - 2x^3 - 3x^2 + 8x - 4$

$$\begin{array}{r|rrrrr} 1 & 1 & -2 & -3 & 8 & -4 \\ & & 1 & -1 & -4 & 4 \\ \hline & 1 & -1 & -4 & 4 & 0 \end{array} \qquad (x-1)(x^3 - x^2 - 4x + 4)$$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -4 & 4 \\ & & 1 & 0 & -4 \\ \hline & 1 & 0 & -4 & 0 \end{array} \qquad \begin{array}{l} (x-1)^2(x^2-4) \\ \boxed{(x-1)^2(x+2)(x-2)} \end{array}$$

Deg $= 4$

$\uparrow \quad \uparrow$



WS: #5-9 list root (p/q's)

#18, 21, 24, 27

~~factor completely~~ +

Graph