Note that we also could have solved the equation in Example 8 by first writing it as

$$\tfrac{4}{3}\pi r^3 + 4\pi r^2 - 100 = 0$$

and then finding the x-intercept of the function $y = \tfrac{4}{3}\pi x^3 + 4\pi x^2 - 100$.

4.4 EXERCISES

CONCEPTS

1. If the polynomial function

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

has integer coefficients, then the only numbers that could possibly be rational zeros of P are all of the form $\frac{p}{q}$, where p is a factor of _____ and q is a factor of _____. The possible rational zeros of $P(x) = 6x^3 + 5x^2 - 19x - 10$ are _____.

2. Using Descartes' Rule of Signs, we can tell that the polynomial $P(x) = x^5 - 3x^4 + 2x^3 - x^2 + 8x - 8$ has _____, _____, or _____ positive real zeros and _____ negative real zeros.

3. *True or false?* If c is a real zero of the polynomial P, then all the other zeros of P are zeros of $P(x)/(x - c)$.

4. *True or false?* If a is an upper bound for the real zeros of the polynomial P, then $-a$ is necessarily a lower bound for the real zeros of P.

SKILLS

5–10 ▫ List all possible rational zeros given by the Rational Zeros Theorem (but don't check to see which actually are zeros).

5. $P(x) = x^3 - 4x^2 + 3$

6. $Q(x) = x^4 - 3x^3 - 6x + 8$

7. $R(x) = 2x^5 + 3x^3 + 4x^2 - 8$

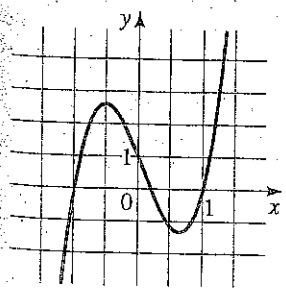
8. $S(x) = 6x^4 - x^2 + 2x + 12$

9. $T(x) = 4x^4 - 2x^2 - 7$

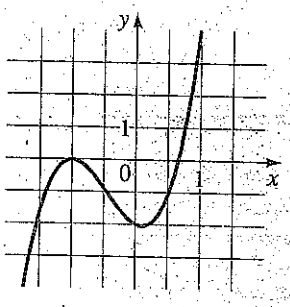
10. $U(x) = 12x^5 + 6x^3 - 2x - 8$

11–14 ▫ A polynomial function P and its graph are given. **(a)** List all possible rational zeros of P given by the Rational Zeros Theorem. **(b)** From the graph, determine which of the possible rational zeros actually turn out to be zeros.

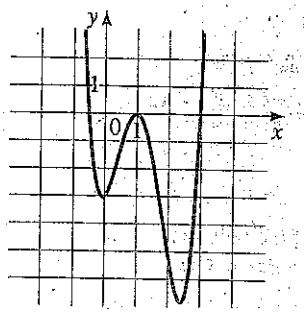
11. $P(x) = 5x^3 - x^2 - 5x + 1$



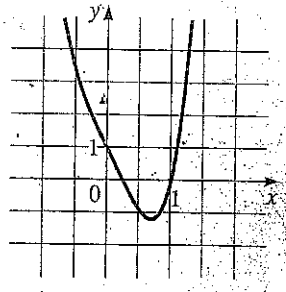
12. $P(x) = 3x^3 + 4x^2 - x - 2$



13. $P(x) = 2x^4 - 9x^3 + 9x^2 + x - 3$



14. $P(x) = 4x^4 - x^3 - 4x + 1$



15–44 ▫ Find all rational zeros of the polynomial, and write the polynomial in factored form.

15. $P(x) = x^3 + 3x^2 - 4$

16. $P(x) = x^3 - 7x^2 + 14x - 8$

17. $P(x) = x^3 - 3x - 2$

18. $P(x) = x^3 + 4x^2 - 3x - 18$

19. $P(x) = x^3 - 6x^2 + 12x - 8$

20. $P(x) = x^3 - x^2 - 8x + 12$

21. $P(x) = x^3 - 4x^2 + x + 6$

22. $P(x) = x^3 - 4x^2 - 7x + 10$

23. $P(x) = x^3 + 3x^2 - x - 3$

24. $x^3 - 4x^2 - 11x + 30$

27. $x^4 + 6x^3 + 7x^2 - 6x - 8$

4.4 5-9 list, 18, 21, 24, 27 homework

⑤ $p(x) = x^3 - 4x^2 + 3$

$1, -1, 3, -3$

⑥ $Q(x) = x^4 - 3x^3 - 6x + 8$

$\pm(1, 2, 4, 8)$

⑦ $r(x) = 2x^5 + 3x^3 + 4x^2 - 8$

$\frac{\text{Factors } 8}{\text{Factors } 2} \quad \pm\frac{(1,2,4,8)}{1,2} = \pm(1, 2, 4, 8, \frac{1}{2})$

⑧ $s(x) = 6x^4 - x^2 + 2x + 12$

$\frac{\text{Factors } 12}{\text{Factors } 6} = \pm\frac{(1,2,3,4,6,12)}{1,2,3,6} = \pm(1, 2, 3, 4, 6, 12, \frac{1}{2}, \frac{3}{2}, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{1}{6})$

⑨ $T(x) = 4x^4 - 2x^2 - 7$

$\frac{\text{Factors } 7}{\text{Factors } 4} = \frac{\pm(1,7)}{1,2,4} = \pm(1, 7, \frac{1}{2}, \frac{7}{2}, \frac{1}{4}, \frac{7}{2})$

⑱ $p(x) = x^3 + 4x^2 - 3x - 18$ $\quad \pm 1, 2, 3, 6, 9, 18$

2) $\begin{array}{r|rrrr} & 1 & 4 & -3 & -18 \\ \downarrow & & 2 & 12 & 18 \\ \hline & 1 & 6 & 9 & 0 \end{array}$

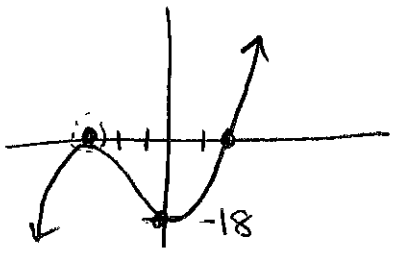
$(x-2)(x^2 + 6x + 9)$

$y = (x-2)(x+3)^2$

x int $2, -3$

y int -18

Deg 3



㉑ $p(x) = x^3 - 4x^2 + x + 6$ $\quad \pm 1, 2, 3, 6$

-1) $\begin{array}{r|rrrr} & 1 & -4 & 1 & 6 \\ \downarrow & & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$

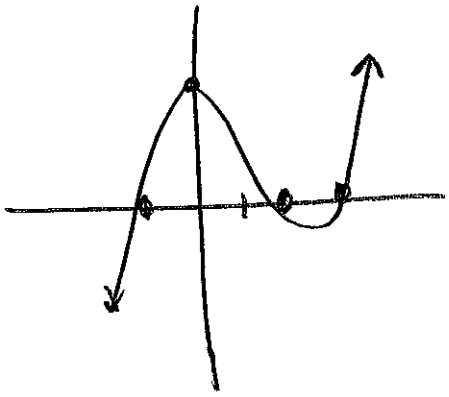
$(x+1)(x^2 - 5x + 6)$

$y = (x+1)(x-3)(x-2)$

x int $-1, 3, 2$

y int 6

Deg 3



(24) $p(x) = x^3 - 4x^2 - 11x + 30$

$\pm(1, 2, 3, 5, 6, 10, 15, 30)$

2 | 1 -4 -11 30

↓ 2 -4 30

1 -2 -15 0

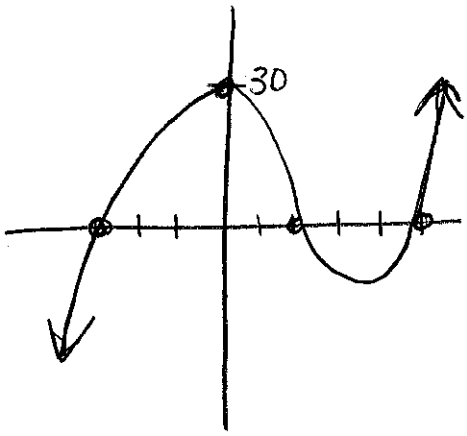
$(x-2)(x^2-2x-15)$

$y = (x-2)(x-5)(x+3)$

Deg 3

Xint 2, 5, -3

yint 30



(27) $p(x) = x^4 + 6x^3 + 7x^2 - 6x - 8$

$\pm(1, 2, 4, 8)$

$y = (x-1)(x+1)(x+4)(x+2)$

1 | 1 6 7 -6 -8

↓ 1 7 14 8

1 7 14 8 0

Deg 4

Xint 1, -1, -4, -2

yint -8

-1 | 1 7 14 8

↓ -1 -6 -8

1 6 8 0