Into to Quadratics

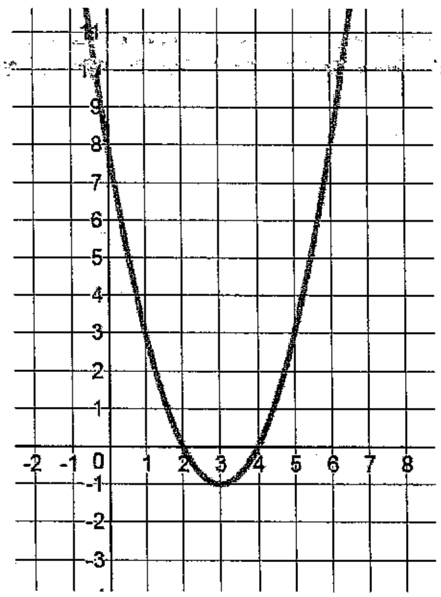
Key Features of a Quadratic Function

Read and Try

In the last chapter, you learned that if the highest degree in a polynomial expression is 2, the expression is called <u>quadratic</u>. This entire chapter is about graphing and interpreting quadratic functions. In general, a quadratic function is in the form: $y = ax^2 + bx + c$. The graph of every quadratic function is a U-shaped curve called a <u>parabola</u>

Look carefully at the graph of a quadratic function:

$$y = x^2 - 6x + 8$$



Note that in this equation, if you examine the form

$y = ax^2 + bx + c$, you can see that

$a = 1$, $b = -6$, and $c = 8$

Now look more carefully at the following characteristics:

<table>
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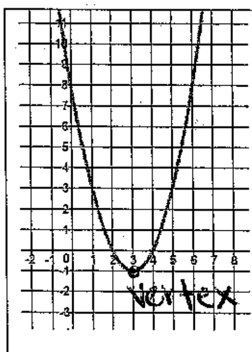
<td>
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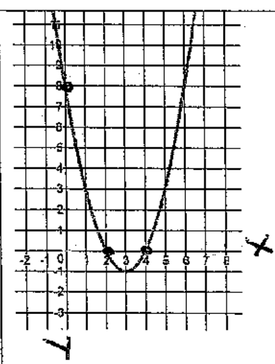
<td>
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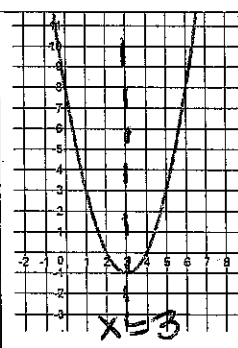
<td>
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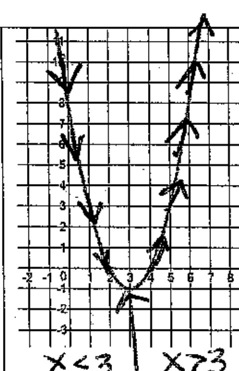
<td>
</td>
</tr>
<tr>
<td>The parabola is upward facing

The vertex is at (3, -1) and is a minimum value</td>
<td>The x-intercepts are 2 and 4

The y-intercept is at 8</td>
<td>The axis of symmetry is the line $x = 3$

Vertical line passing through vertex</td>
<td>The parabola is decreasing when $x < 3$

The parabola is increasing when $x > 3$</td>
</tr>
<tr>
<td colspan="4">

- Because any value of x may be input into the equation, the <u>domain</u> of the function is all real numbers
- Because the graph has a low point when $y = -1$, and then increases forever, the <u>range</u> of the function is $y \geq -1$

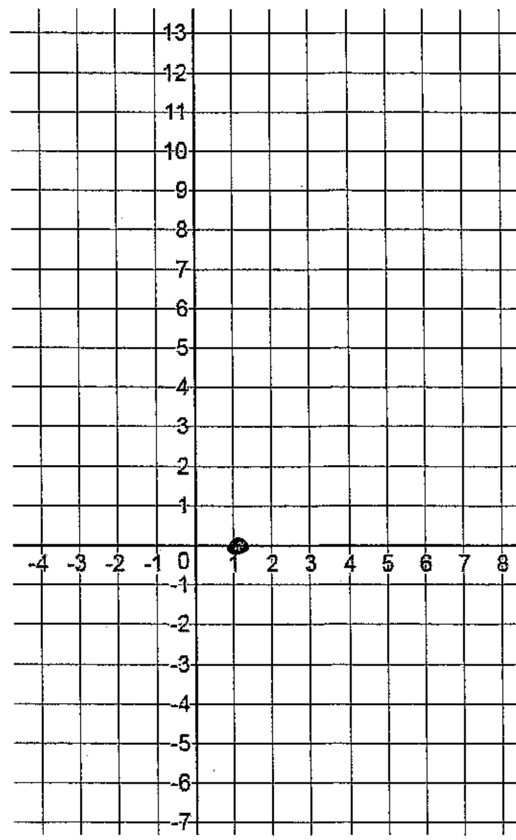
</td>
</tr>
</table>

For the next two problems, you will find the graph of the parabolas by first completing the table. After you have completed the table and graphed, you may want to check your graph using a graphing calculator. Note the important characteristics. Need help? Check out the previous example!

1. Graph $y = 2x^2 - 8x + 6$

x	$y = 2x^2 - 8x + 6$	Coordinate:
0		(,)
1	$2(1)^2 - 8(1) + 6 = 0$	(1 , 0)
2		(,)
3		(,)
4		(,)

Is the parabola upward or downward facing?



Where is the vertex?

Is the vertex a minimum or a maximum?

What are the x-intercept(s)?

What is the y-intercept?

What is the axis of symmetry? x = ________

The parabola is decreasing when x < ______

The parabola is increasing when x > ______

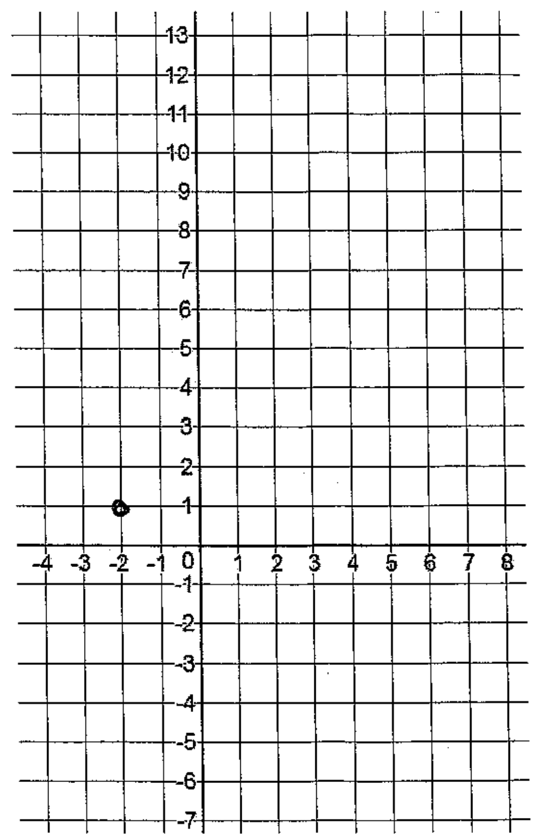
Because *any* x-value could be input into the equation, the domain is ____________________________

Because the graph has a low point when y = ______, and then increases forever, the range of the function is ________

2. Graph $y = -x^2 - 4x - 3$

x	$y = -x^2 - 4x - 3$	Coordinate:
-4		(,)
-3		(,)
-2	$-(-2)^2-4(-2)-3=1$	(-2 , 1)
-1		(,)
0		(,)

Is the parabola upward or downward facing?



Where is the vertex?

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What are the x-intercept(s)?

What is the y-intercept?

What is the axis of symmetry? x = __________

The parabola is increasing when x < ________

The parabola is decreasing when x > ________

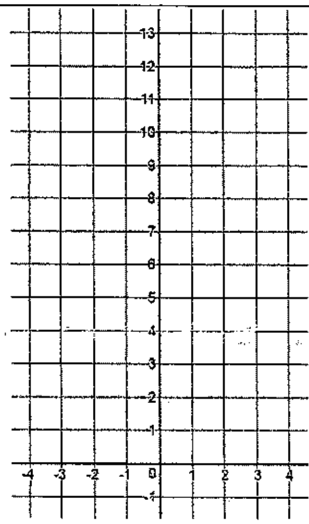
Because *any* x-value could be input into the equation, the domain is ________________________________

Because the graph has a high point when y = _______, and then decreases forever, the range of the function is $y \leq$ __________

The last few should be a little quicker to graph, because they are of the form $y = ax^2$. This means there is no vertical or horizontal shift—these will all have a vertex located at the origin.

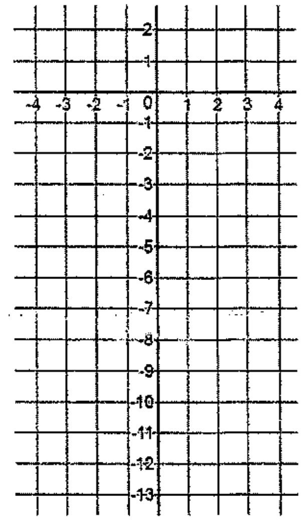
1. $y = 3x^2$

x	y
-2	
-1	
0	
1	
2	



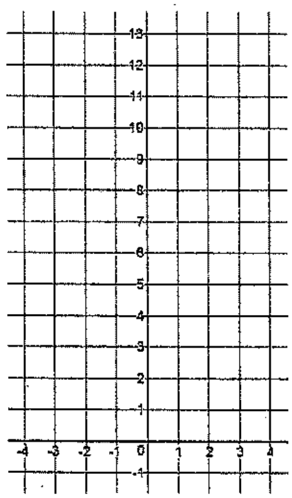
2. $y = -3x^2$

x	y
-2	
-1	
0	
1	
2	



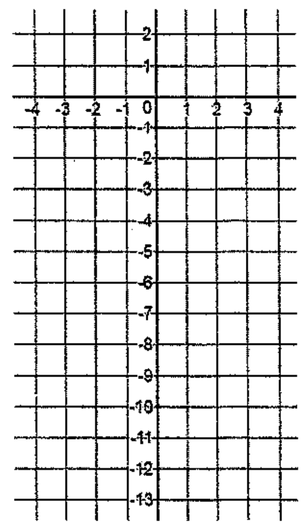
3. $y = x^2$

x	y
-2	
-1	
0	
1	
2	



4. $y = -x^2$

x	y
-2	
-1	
0	
1	
2	



Now it is time to make some conclusions.

1. Some of the parabolas were upward facing, and some were downward facing. What in the equation do you believe would cause a parabola to be downward facing?

2. Some of the parabolas were narrower than other parabolas. What in the equation do you believe would cause a parabola to be narrower than $y = x^2$?

3. Based on your conclusions, make some predictions: The graph of $y = -5x^2$ will be ________________ (upward or downward) facing, and will be ____________________ (more or less) narrow than the graph of $y = x^2$. If you have a graphing calculator, see if your predictions were correct!

Intro to Quadratics Key

Key Features of a Quadratic Function

~~Read and Try~~

In the last chapter, you learned that if the highest degree in a polynomial expression is 2, the expression is called <u>quadratic</u>. This entire chapter is about graphing and interpreting quadratic functions. In general, a quadratic function is in the form: $y = ax^2 + bx + c$. The graph of every quadratic function is a U-shaped curve called a <u>parabola</u>

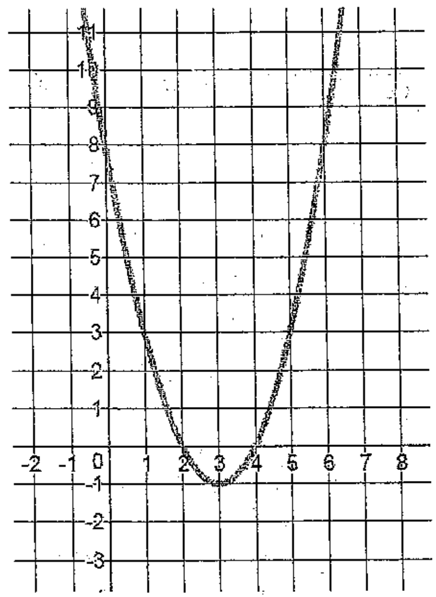
Look carefully at the graph of a quadratic function:

$$y = x^2 - 6x + 8$$

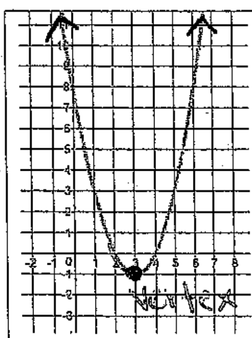
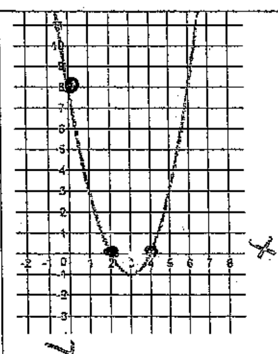
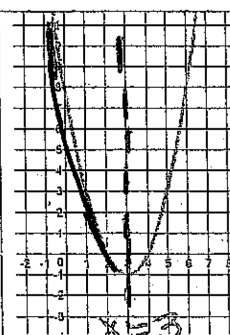
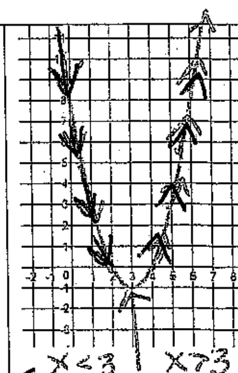
Note that in this equation, if you examine the form

$y = ax^2 + bx + c$, you can see that

$a = 1$, $b = -6$, and $c = 8$



Now look more carefully at the following characteristics:

			
The parabola is upward facing The <u>vertex</u> is at (3, -1) and is a *minimum* value	The x-intercepts are 2 and 4 The y-intercept is at 8	The axis of symmetry is the line x = 3 Vertical line passing through vertex	The parabola is decreasing when $x < 3$ The parabola is increasing when $x > 3$

- Because *any* value of x may be input into the equation, the <u>domain</u> of the function is *all real numbers*
  left right
- Because the graph has a low point when $y = -1$, and then increases forever, the <u>range</u> of the function is $y \geq -1$

For the next two problems, you will find the graph of the parabolas by first completing the table. After you have completed the table and graphed, you may want to check your graph using a graphing calculator. Note the important characteristics. Need help? Check out the previous example!

1. Graph $y = 2x^2 - 8x + 6$

x	$y = 2x^2 - 8x + 6$	Coordinate:
0	$2(0)^2-8(0)+6=6$	(0 , 6)
1	$2(1)^2-8(1)+6=$	(1 , 0)
2		(2 , -2)
3		(3 , 0)
4		(4 , 6)

Is the parabola upward or downward facing?

upward

Where is the vertex?

(2,-2)

Is the vertex a minimum or a maximum?

minimum

What are the x-intercept(s)?

1 and 3

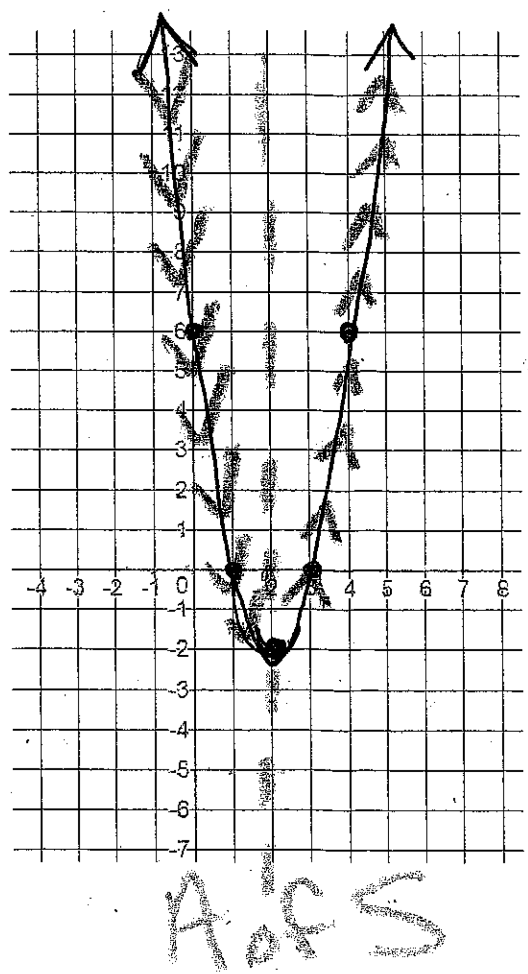
What is the y-intercept? 6

What is the axis of symmetry? x = 2

The parabola is decreasing when x < 2

The parabola is increasing when x > 2

Because *any* x-value could be input into the equation, the domain is all real #'s



Because the graph has a low point when y = -2, and then increases forever, the range of the function is $y \geq -2$

2. Graph $y = -x^2 - 4x - 3$

x	$y = -x^2 - 4x - 3$	Coordinate:
-4	$-(-4)^2 - 4(-4) - 3$	(-4 , -3)
-3	$-(-3) - 4(-3) - 3$	(-3 , 0)
-2	$-(-2) - 4(-2) - 3$	(-2 , 1)
-1	$-(-1) - 4(-1) - 3$	(-1 , 0)
0	$-(0) - 4(0) - 3$	(0 , -3)

Is the parabola upward or downward facing?

downward

Where is the vertex? (-2, 1)

Is the vertex a minimum or a maximum?

What are the x-intercept(s)?

-3 and -1

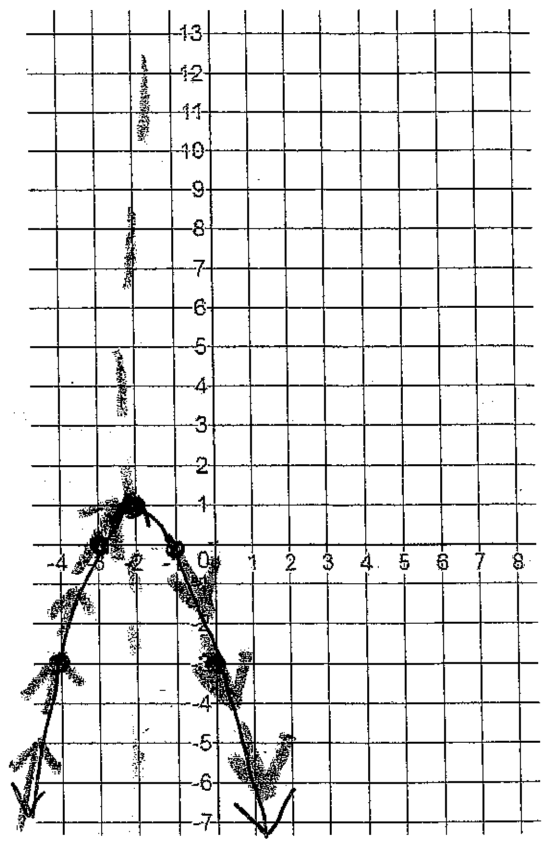
What is the y-intercept? -3

What is the axis of symmetry? x = -2

The parabola is increasing when x < -2

The parabola is decreasing when x > -2

Because *any* x-value could be input into the equation, the domain is all real #s

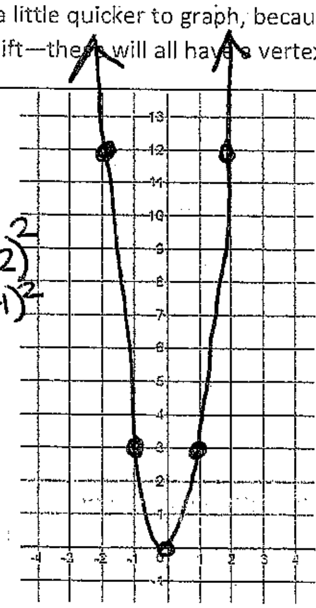


Because the graph has a high point when y = 1, and then decreases forever, the range of the function is $y \leq$ 1

The last few should be a little quicker to graph, because they are of the form $y = ax^2$. This means there is no vertical or horizontal shift—these will all have a vertex located at the origin.

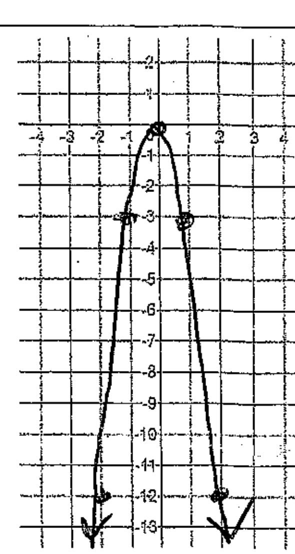
1. $y = 3x^2$

x	y	
-2	12	$3(-2)^2$
-1	3	$3(-1)^2$
0	0	
1	3	
2	12	



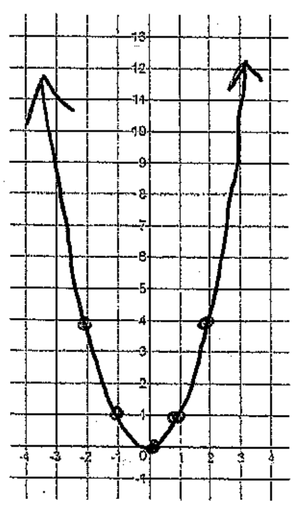
2. $\boxed{y = -3x^2}$

x	y
-2	-12
-1	-3
0	0
1	-3
2	-12



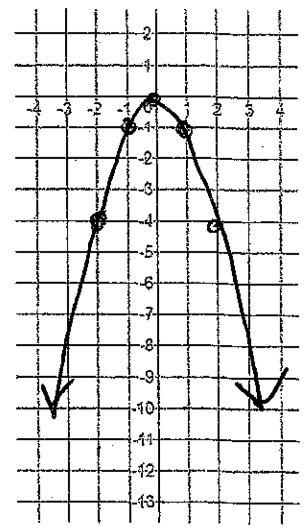
3. $y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4



4. $\boxed{y = -x^2}$

x	y
-2	-4
-1	-1
0	0
1	-1
2	-4



Now it is time to make some conclusions.

1. Some of the parabolas were upward facing, and some were downward facing. What in the equation do you believe would cause a parabola to be downward facing?

   negative in front of x^2

2. Some of the parabolas were narrower than other parabolas. What in the equation do you believe would cause a parabola to be narrower than $y = x^2$?

   The coeffient of x^2 being greater than 1

3. Based on your conclusions, make some predictions: The graph of $y = -5x^2$ will be downward (upward or downward) facing, and will be more (more or less) narrow than the graph of $y = x^2$. If you have a graphing calculator, see if your predictions were correct!